Symplectic Dynamics in Montevideo

Abstracts

Research talks.

- **Patrice Le Calvez** (Sorbonne), *Periodic orbits of area preserving surface homeomorphisms with rational direction.*

  In a joint work with Pierre-Antoine Guihéneuf and Alejandro Passeggi we will explain why an area preserving homeomorphism of a closed surface isotopic to the identity that has a rotation vector with a rational direction has infinitely many periodic orbits (result independently proved by Rohil Prasad). More precisely, we will explain how associate to every non trivial ergodic measure an "interval" of periodic orbits.

- **Julian Chaidez** (USC), *From Gromov-Witten theory to dynamics.*

  A general philosophy due to Joel Fish and Helmut Hofer states that symplectic manifolds with sufficiently rich Gromov-Witten theory should contain hypersurfaces with rich dynamics. In this talk, we will discuss many recent results that represent specific instances of this philosophy. In low dimensions, where Gromov-Witten (and related Gromov-Taubes) invariants are more well understood, strong and general results like the smooth closing lemma have been proven. The higher dimensional setting is much more mysterious, and is the subject of ongoing research.

  This talk is based on joint work with Shira Tanny (and the work of others).

- **Urs Frauenfelder** (Augsburg), *Spectral Jumps in Tate Rabinowitz Floer homology.*

  This is joint work with Kai Cieliebak. We are looking for n interacting particles. For these we are introducing the delayed Rabinowitz action functional which is invariant under the action on an n-dimensional torus, which appears by reparametrizing the time of each particle individually. The corresponding Tate chain complex admits a double filtration. This leads to the phenomenon of spectral jumps. We are relating these to EBK quantization.

- **Sonja Hohloch** (Antwerp), *Symplectic geometry around (hyper)semitoric integrable systems.*

  Since completely integrable systems naturally induce a (singular) Lagrangian fibration and thus have a natural link to symplectic topology,

there are many different aspects under which intgrable systems can be studied.

In this talk, we focus on completely integrable Hamiltonian systems on 4-dimensional symplectic manifolds that have an underlying effective Hamiltonian S^1-action. A very special example are toric systems, more general classes of systems are so-called semitoric and hypersemitoric ones. We will relate them to each other and present interesting new symplectic and dynamical features.

- **Surena Hozoori** (Rochester), *Symplectic geometry of Anosov 3-flows.*

  Since their introduction in the early 1960s, Anosov flows have defined an important class of dynamics, thanks to their many interesting chaotic features and rigidity properties. Moreover, their topological aspects have been deeply explored, in particular in low dimensions, thanks to the use of foliation theory in their study. Although the connection of Anosov flows to contact and symplectic geometry was noted in the mid 1990s by Mitsumatsu and Eliashberg-Thurston, such interplay has been left mostly unexplored. I will present some recent results on the contact and symplectic geometric theory of Anosov flows in dimension 3. Time permitting, various related topics will be discussed, including the interplay of Anosov flows with Reeb dynamics, Liouville geometry and surgery theory, the presence of invariant volume forms, etc.

- **Umberto Hryniewicz** (Aachen), *Proof of Hofer-Wysocki-Zehnder's two or infinity conjecture.*

  We prove that every Reeb flow on a closed connected three-manifold has either two or infinitely many periodic orbits, if the first Chern class of the associated contact structure is torsion. This result implies that every Finsler metric on a closed surface has either two or infinitely many closed geodesics.

- **Levin Maier** (Heidelberg), *Hofer–Zehnder capacity of disc tangent bundles of lens spaces.*

  We compute the Hofer–Zehnder capacity of disc tangent bundles of certain lens spaces with respect to the round metric. Interestingly we find that the Hofer–Zehnder capacity does not see the covering, i.e. the capacity of the disc tangent bundle of the lens space coincides with the capacity of the disc tangent bundle of the 3-sphere covering it. In particular this gives a first example, where Gromov width and Hofer–Zehnder capacity of a disc tangent bundle disagree. Techniques we use include for the lower bound magnetic billiards and for the upper bound Gromov–Witten invariants. (j.w. Johanna Bimmermann)

- **Miguel Paternain** (UDELAR), *Periodic orbits of Tonelli Lagrangians with prescribed action.*

It is well known that Tonelli Lagrangians have periodic orbits with energy larger than Mañé's critical value. We show that a similar result holds for the action. We show that for every Tonelli Lagrangian there is a real number a_0 such that for every $a > a_0$ the Lagrangian has a periodic orbit with action a. We obtain estimates on the period and energy of such periodic orbits.

- **Vinicius Ramos** (IMPA), *Normalizations of symplectic capacities.*

  Symplectic capacities are a very important tool in the study of symplectic embedding problems. In the definition of capacities, it is often required that they coincide for a ball and a cylinder of the same radius. It is conjectured that this condition determines the value of a capacity for convex sets. However, many capacities do not satisfy these conditions, such the Lagrangian capacity and most of the Ekeland-Hofer and ECH capacities. In this talk, I will discuss other normalization conditions and talk about what is known about whether they characterize capacities in different types of domains. This talk is based on joint work with J. Gutt and M. Pereira.

Mini-courses.

- **Michael Hutchings** (UC Berkeley), *The Weyl law for embedded contact homology.*

  In the first lecture, we will review the definition of embedded contact homology (ECH). Since there will not be nearly enough time to explain everything, participants may find it helpful to consult my "Lecture notes on embedded contact homology". In the second lecture we will discuss ECH spectral invariants, and a "Weyl law" which relates the asymptotics of ECH spectral invariants to contact volume. In the third lecture, we will present applications of the Weyl law to various topics in dynamics of Reeb vector fields.

- **Agustin Moreno** (Heidelberg), *On the three-body problem.*

  The restricted three-body problem is a classical conundrum going back to Newton and Poincaré, concerning the motion of a negligible mass under the gravitational influence of two large masses. An astounding open challenge, it is not only interesting from a theoretical perspective, but also from a practical perspective, as it has applications in astronomy and space mission design.

  In this mini-course, I will revisit this classical problem from the modern vantage point of contact and symplectic geometry. I will touch on open book decompositions, fixed-point theory for Hamiltonian maps, and finite energy foliations. Part of the story will be based on joint work with Otto van Koert (Seoul). Time permitting, I will also discuss ongoing work with Cengiz Aydin (Heidelberg), Urs Frauenfelder (Augsburg), Dayung

Koh (JPL, NASA), and Otto van Koert (Seoul), concerning symplectic methods in space mission design.

- **Ana Rechtman** (Grenoble), *Broken (open) books and Reeb dynamics.*

  Some non-singular vector fields on closed 3-manifolds admit transverse sections, with or without boundary, that allow to study the dynamics of the flow through the first return map. A transverse section is a surface that intersects all the orbits in bounded time. If the section has boundary, it is a Birkhoff section and the vector field is supported by a rational open book decomposition. In this case, we require that the boundary is a collection of periodic orbits of the flow.

  A non-degenerate Reeb vector field on a closed 3-manifold is supported by a broken book decomposition, a generalisation of an open book decomposition. This result has several implication on the dynamics of the flow, in particular from a broken book decomposition one can build an open book decomposition under extra hypothesis on the Reeb vector field.

  In this series of talks, I will give some elements of the construction of a broken book decomposition and concentrate on the consequences that such a structure has on the dynamics of the flow. A particular case of Reeb vector fields are geodesic flows of surfaces, I will recall Birkhoff construction of sections in this case.